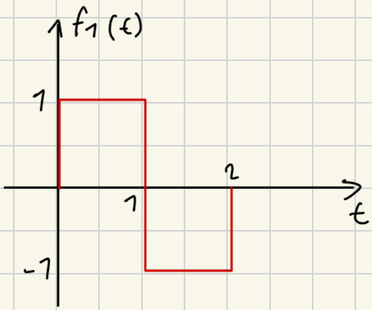
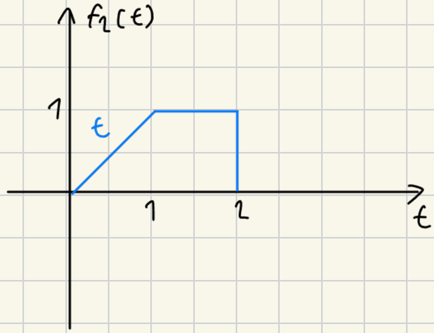
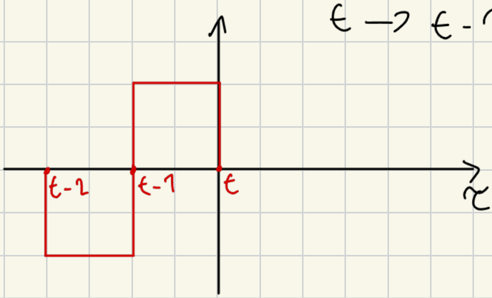


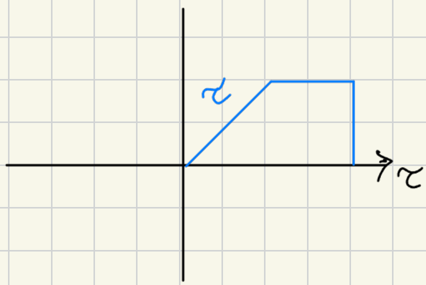


$$g(t) = f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau)\, f_2(t-\tau)\, d\tau = \int_{-\infty}^{\infty} f_2(\tau)\, f_1(t-\tau)\, d\tau$$

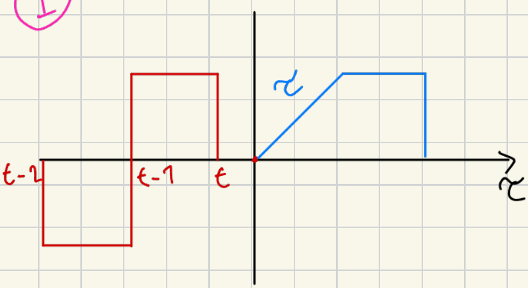
Paso 1 $\longrightarrow$ Invertir y desplazar

$t \rightarrow t - \tau$ $\qquad\qquad$ $t \rightarrow \tau$





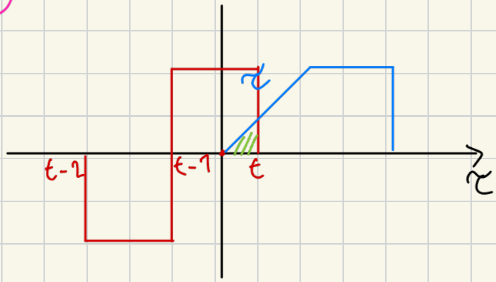
I



Para $t < 0$

$$g(t) = 0$$

II



$0 < t < 1$

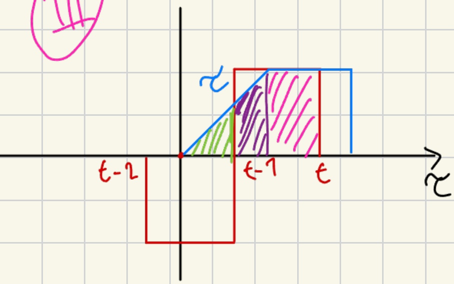
$$g(t) = \int_0^t (1)(\tau)\,d\tau = \frac{\tau^2}{2}\Big|_0^t$$

$$\boxed{g(t) = \frac{t^2}{2}}$$

$\xrightarrow{0} 0$

$\xrightarrow{1} \frac{1}{2}$

III



$1 < t < 2$

$$g(t) = \int_0^{t-1} (-1)(\tau)\,d\tau + \int_{t-1}^{1} (1)(\tau)\,d\tau + \int_1^t (1)(1)\,d\tau$$

$$= -\frac{\tau^2}{2}\Big|_0^{t-1} + \frac{\tau^2}{2}\Big|_{t-1}^{1} + \tau\Big|_1^t$$

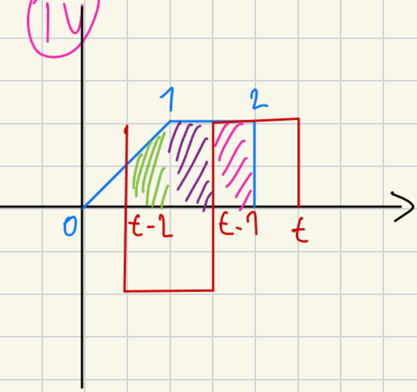
$$= -\frac{(t-1)^2}{\cancel{2}} + 0 + \frac{1}{2} - \frac{(t-1)^2}{\cancel{2}} + t - 1$$

$$\boxed{= -(t-1)^2 + t - \frac{1}{2}}$$

$\xrightarrow{t=2} \frac{1}{2}$

$\xrightarrow{t=1} \frac{1}{2}$

IV



$$\overset{+1}{1} < \overset{+1}{t-1} < \overset{+1}{2}$$
$$2 < t < 3$$

$$g(t) = \int_{t-2}^{1} (-1)(\tau)\,d\tau + \int_{1}^{t-1} (-1)(1)\,d\tau + \int_{t-1}^{2} (1)(1)\,d\tau$$

$$= -\frac{\tau^2}{2}\Big|_{t-2}^{1} - \tau\Big|_{1}^{t-1} + \tau\Big|_{t-1}^{2}$$

$$= -\frac{1}{2} + \frac{(t-2)^2}{2} - (t-1) + 1 + 2 - (t-1)$$

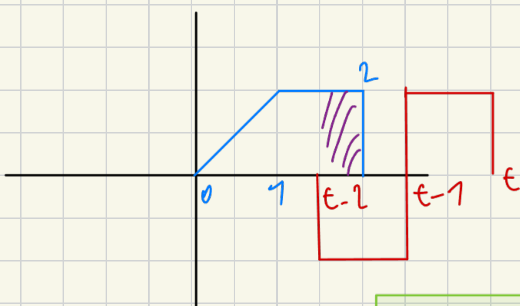
$$\boxed{= \frac{(t-2)^2}{2} - 2(t-1) + \frac{5}{2}}$$

$t=2 \rightarrow \frac{1}{2}$

$t=3 \rightarrow -1$

V

$$1 < t-2 < 2$$
$$3 < t < 4$$



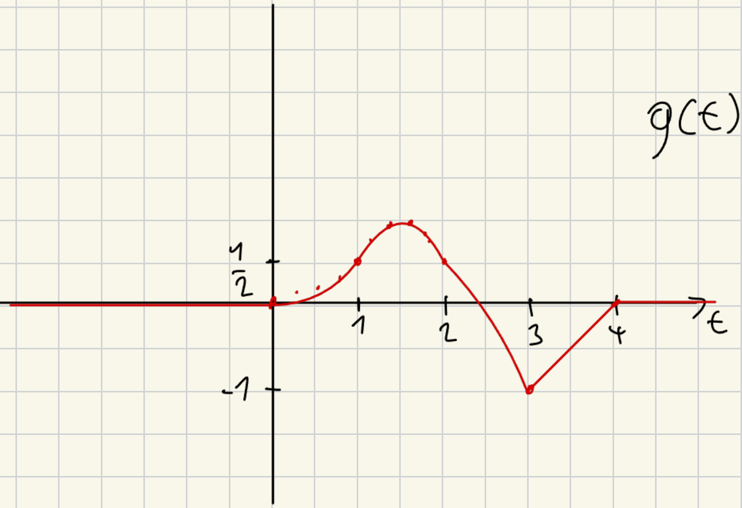
$$g(t) = \int_{t-2}^{2} (1)(-1)\,d\tau = -\tau\Big|_{t-2}^{2}$$

$$\boxed{g(t) = -2 + (t-2) = t-4}$$

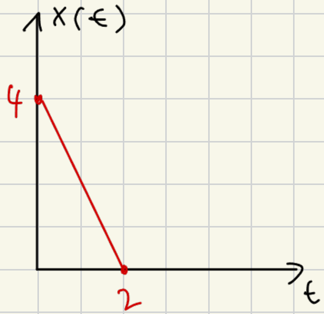
$t=3 \rightarrow -1$

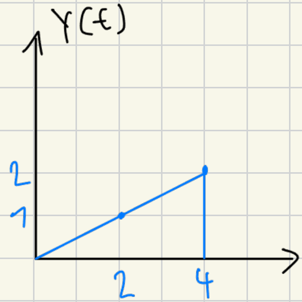
$t=4 \rightarrow 0$

VI Para $t > 4$ no hay solape $\therefore g(t) = 0$



$$g(t) = \begin{cases} \frac{t^2}{2}, & 0 < t < 1 \\ -(t-1)^2 + t - \frac{1}{2}, & 1 < t < 2 \\ \frac{(t-2)^2}{2} - 2(t-1) + \frac{5}{2}, & 2 < t < 3 \\ t-4, & 3 < t < 4 \\ 0, & \text{eoc} \end{cases}$$



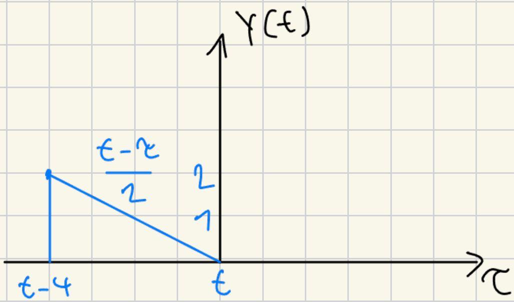


$x(t) = -2(t-2)$

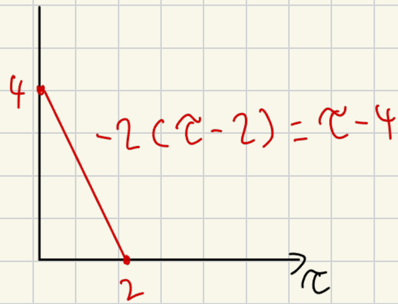
$Y(t) = \frac{t}{2}$

Invertir y desplazar

$t \to t - \tau$



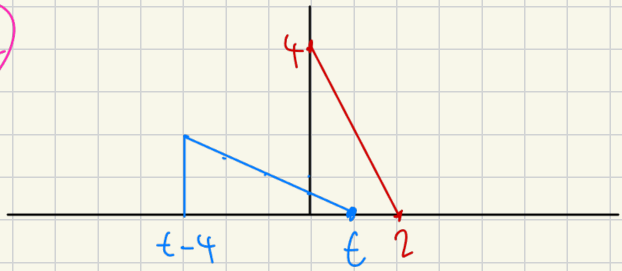
$t \to \tau$



$$g(t) = f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t-\tau)\,d\tau = \int_{-\infty}^{\infty} f_2(\tau) f_1(t-\tau)\,d\tau$$

- Para $t < 0$, $g(t) = 0$

I



$0 < t < 2$

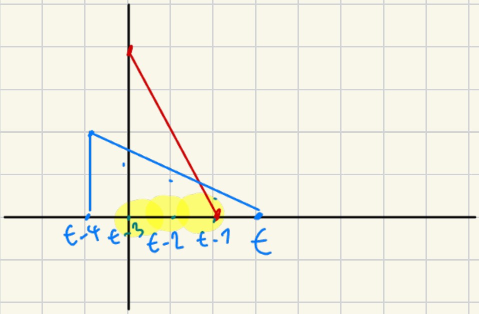
$$g(t) = \int_0^t \frac{1}{\cancel{2}}(t-\tau)(-\cancel{2}(\tau-2)) = \int_0^t (t-\tau)(2-\tau)\,d\tau$$

$$= \int_0^t (\tau^2 - \tau t - 2\tau + 2t)\,d\tau = \frac{\tau^3}{3} - \frac{t\tau^2}{2} - \tau^2 + 2t\tau \Big|_0^t$$

$$= \frac{t^3}{3} - \frac{t^3}{2} - t^2 + 2t^2$$

$$= \boxed{-\frac{t^3}{6} + t^2} \quad \xrightarrow{t=0} 0 \qquad \xrightarrow{t=2} 8/3$$

II



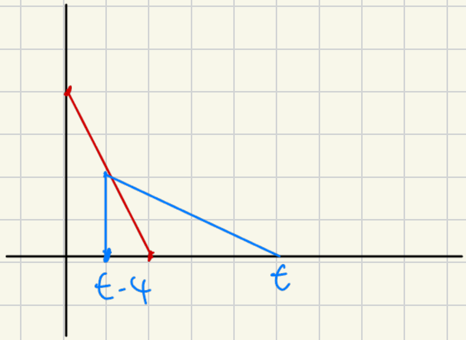
$2 < t < 4$

$$g(t) = \int_0^2 (t-\tau)(2-\tau)\,d\tau$$

$$= \frac{\tau^3}{3} - \frac{t\tau^2}{2} - \tau^2 + 2t\tau \Big|_0^2$$

$$= \frac{8}{3} - 2t - 4 + 4t = \boxed{2t - 4/3} \quad \xrightarrow{t=2} 8/3 \qquad \xrightarrow{t=4} 20/3$$

III



$0 < t-4 < 2$

$4 < t < 6$

$$g(t) = \int_{t-4}^{2} (t-\tau)(2-\tau)\,d\tau$$

$$= \frac{\tau^3}{3} - \frac{t\tau^2}{2} - \tau^2 + 2t\tau \Big|_{t-4}^{2}$$

$$= 2t - \frac{4}{3} - \frac{(t-4)^3}{3} + t\frac{(t-4)^2}{2} + (t-4)^2 - 2t(t-4)$$

• $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$$= 2t - \frac{4}{3} - \left[\frac{t^3}{3} + \frac{\cancel{3}t^2(-4)}{\cancel{3}} + \frac{\cancel{3}t(-4)^2}{\cancel{3}} - \frac{64}{3}\right] + t\left[\frac{t^2 - 8t + 16}{2}\right]$$

$$+ t^2 - \cancel{8t} + 16 - 2t^2 + \cancel{8t}$$

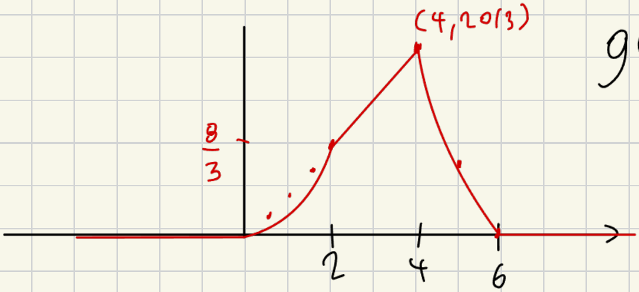
$$= 2t - \frac{4}{3} - \frac{t^3}{3} + \cancel{4t^2} - 16t + \frac{64}{3} + \frac{t^3}{2} - \cancel{4t^2} + 8t + t^2 + 16 - 2t^2$$

$$= \boxed{\frac{t^3}{6} - t^2 - 6t + 36}$$

$t=4 \rightarrow 20/3$

$t=6 \rightarrow 0$

• $t > 6$, $g(t) = 0$



$$g(t) = \begin{cases} -\frac{t^3}{6} + t^2, & 0 < t < 2 \\ 2t - \frac{4}{3}, & 2 < t < 4 \\ \frac{t^3}{6} - t^2 - 6t + 36, & 4 < t < 6 \\ 0, & eoc \end{cases}$$